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PRECAUTIONARY SAVING, INEQUALITY AND FISCAL POLICY: A HANK MODEL

Abstract: *We construct a Hank model with unemployment risk to study the effects of fiscal policies on reducing inequality in Chinese residents and find that: (1) Due to savings decisions and liquidity constraints, the increase in inequality will increase the total savings rate, which is not conducive to economic transformation. (2) The impact of income tax on inequality depends on its progressivity degree. Reducing proportional tax can improve inequality, and general transfer payments have no significant effect on improving inequality. However, increasing unemployment and poverty subsidies can reduce inequality, increase the consumption of residents with a high MPC and achieve a balance between fairness and efficiency. (3) The effect of monetary policy depends on the response of fiscal policy, and the improvement of inequality is conducive to smoothing the transmission mechanism of monetary policy and stimulating consumption. In terms of algorithms, we also propose a hybrid algorithm combining projection and perturbation algorithms for heterogeneous agent models.*

Keywords*: Inequality, precautionary savings, marginal propensity to consume, heterogeneity, computational economics.*

JEL Classification: C11, C51, C63, C61, D52

1. Introduction

In recent years, the increasing inequality of wealth, income and consumption has caused extensive concern among scholars (Piketty et al., 2003). Since the reform and opening up, China has achieved a miracle of rapid economic development and long-term social stability. However, the rising Gini coefficient reminds us of the deteriorating problem of residents' wealth distribution and income

DOI: 10.24818/18423264/55.1.21.04

inequality. Figure1 shows residents' income and wealth inequality from 1978 to 2015 in China.

As an important component of macroeconomic policy, fiscal policy has a significant redistribution effect among the income and wealth distribution of residents. How to make good use of fiscal policy to improve the inequality under the consideration of both efficiency and fairness has become an important theoretical and practical issue.

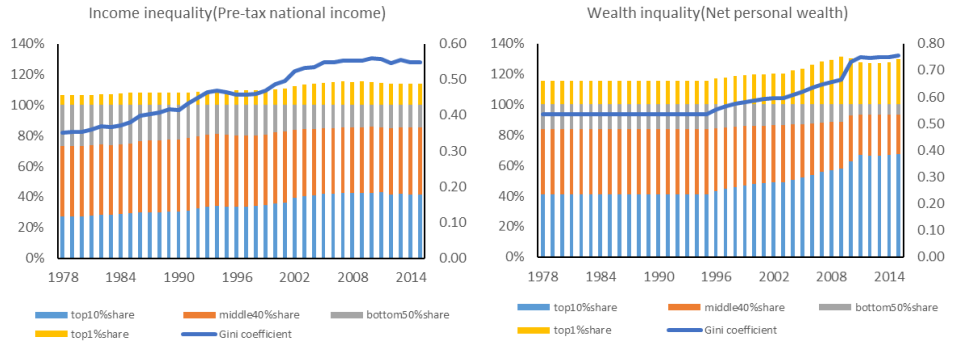

Figure 1. Residential income quantile, wealth quantile and Gini coefficient

In theory, the IS / LM model, the real business cycle theory and the New Keynesian model generally focus on aggregates. Neoclassical growth models rarely involve income and wealth inequality. The representative agent and the complete market assumption make the initial wealth distribution permanent and do not affect economic stability, so the income and wealth distribution cannot be formed endogenously. The overlapping generation model (OLG) introduces inequality into the model by setting two generations. Although the wealth distribution in the life cycle can better meet the inverted U-shaped characteristics in the empirical study, the fitting of consumption data is poor.

Subsequently, the representative agent model begins to absorb the heterogeneity observed from the micro data, and endogenously generate wealth distribution under individual idiosyncratic risk. As a pioneering work, Aiyagari et al (1994) endogenize labor supply deciscion and exogenize family income by establishing an incomplete market, which can provide a preliminary analysis of income distribution and welfare. On the basis of Aiyagari's model, Krusell and Smith (1998) add exogenous shocks to study the relationship between income distribution and economic aggregates. Recently, the interactive relationship between inequality and macroeconomics has become a hot topic, not only because of richer data, the better algorithms and computing power, and more importantly, after the financial crisis, the worsening inequality produces huge impact on economy. The heterogeneous agent model emphasizes heterogeneity in the residential sector, corporate sector, and financial intermediaries. It can endogenize agent's asset distribution and establishe a rich interactive relationship between inequality and macro variables, which can deeply analyze the differences between individuals and the mechanism of influence between individuals and totals. The main perspectives of the heterogeneous agent model to study wealth distribution include special income

DOI: 10.24818/18423264/55.1.21.04

risks, entrepreneurial risks, differences in household discount rates (Krusell and Smith, 1998), the difference in utility of assets held by households (Cagetti and De Nardi, 2006), financial services and financial institutions (Heathcote et al., 2014), etc. In addition, some scholars endogenize income through labor skills, efforts or career choices, human capital investment (Guvenen et al., 2013), etc.

A new framework HANK is emerging recently that combines heterogeneous agents and New Keynesian to study inequality and policy effects (McKay et al., 2016; Ravn and Sterk, 2017; Kaplan et al., 2018; Auclert, 2019). In a Hank model, there are differences in asset liquidity (Kaplan, 2018; Bayer et al., 2019), nominal real value of assets (Auclert, 2019), marginal consumption propensity (Kaplan et al., 2018), borrowing constraints (Acemoglu et al., 2018), etc. Kaplan and Violante (2014) builds a heterogeneous household in portfolio of liquidity, proving that fiscal policy stimulus is better in a mild recession. McKay and Reis (2016) study the taxation's automatic stabilizer effect with the economic cycle model of an incomplete market. In their Hank model, due to the difference in assets and income of heterogeneous agents, the higher marginal consumption propensity of low-income groups produces higher multiplier effect. Kaplan et al. (2018) analyzes the redistribution effect of monetary policy under a Hank model. They believe that the representative agent setting of the DSGE model is too dependent on the intertemporal substitution effect, but under an incomplete market model, monetary policy can play a better role by affecting labor demand rather than intertemporal substitution effect. Not only that, because there is no Ricardo equivalence in government financing, fiscal policy plays a decisive role in the entire model.

Research on fiscal policy and income inequality in China is mainly empirical, from the perspective of marginal consumption propensity, liquidity constraints, and income redistribution, etc. These empirical conclusions are not consistent. Theoretical model studies are few. Bai Zhonglin et al. (2019) studied the income distribution effect of fiscal policy from the perspective of development imbalance by constructing a DSGE model of high and low income heterogeneous households.

Does China's fiscal policy help reduce income and wealth inequality? How does heterogeneity interact with fiscal policy? Overall, these issues have not yet been fully resolved. Exploring these issues not only helps to theoretically clarify the effects and mechanisms of fiscal policy, but also help the Chinese government enhance the effects of macro-control and maintain stable economic growth. The innovations of our study are as follows. First, by introducing individual unemployment risks into the resident sector, we construct a Hank model to study inequality and fiscal policy in China. Second, we break the Ricardo's equivalence hypothesis and analyze the impact of inequality on the effectiveness of monetary and fiscal policies. Third, the Hank model we construct breaks the permanent income hypothesis and can produce an increase in the marginal consumption propensity for transient income increases, which is more in line with empirical results. Technically, we combine the endogenous grid point method on the basis of the mixed algorithm of projection perturbation of reitor (2009), which provides a new idea for solving the infinite dimension calculation of the Hank model.

DOI: 10.24818/18423264/55.1.21.04

2. A model with resident heterogeneity

Based on Aiyagari (1997) and Krusell & Smith (1998), we add the fiscal rules and nominal rigidity to construct a New Keynesian dynamic stochastic general equilibrium benchmark model with household idiosyncratic unemployment risk.

(1) Household

Drawing on the settings of McKay and Reis (2016) and Ravn and Sterk (2017) for the family sector, we assume that the family sector is standardized to 1, and the heterogeneous family individual is labeled as i, $i \in (0, \varpi)$ and survive indefinitely. The heterogeneous family does not hold stocks and capital stocks. They earn income from wages, transfer payments, bond interest and various subsidies. Their savings can be lent to capital goods holders through the bond market to accumulate capital. Heterogeneous families chose the optimal consumption $c_t(i)$, labor hours $l_t(i)$ and bond holdings $b_{t+1}(i)$ to maximize the objective function:

$$\max \sum_{t=0}^{\infty} \beta_1^t \left(\log c_t(i) - \varphi_1 \frac{l_t(i)^{1+\varphi_2}}{1+\varphi_2} \right) \tag{1}$$

$$s.t. \quad \begin{aligned} &(1+\tau_c)\, c_t(i) + \tilde{b}_{t+1}(i)\pi_{t+1} - \tilde{b}_t(i) = \\ &(1-\tau_{i,t}^s(m))[I_{t-1}\tilde{b}_t(i) + e_t(i) w_t l_t(i)] + T_t^b + T^u(e_t(i)) + T^n(e_t(i)) \end{aligned} \tag{2}$$

At the same time, the financial market is incomplete, and Heterogeneous households face lending constraints $\tilde{b}_{t+1}(i) \geq 0$. Therefore, households have precautionary saving motives to protect against overall and individual risks. We assume that heterogeneous families cannot be fully insured and face individual unemployment risks. There are three employment states: employed $e_t(i) = e$, unemployed $e_t(i) = u$, and poor (long-term unemployment) $e_t(i) = n$. The transition probability of heterogeneous family employment status is exogenous, and the transition probability matrix follows the first-order Markov process. If $e_t(i) = u$ or $e_t(i) = n$, then $l_t(i) = 0$, indicating that the unemployed and poor families have no wage income. T_t^b is a general transfer payment to a heterogeneous family, which is irrelevant to employment status. If the heterogeneous family is unemployed $e_t(i) = u$, the family will receive an additional unemployment allowance $T^u(e_t(i))$ with an upper limit $T^u(e_t(i)) \leq \iota \bar{T}^u$. If the residents are in a state of poverty, that is $e_t(i) = n$, the family will receive an additional poverty allowance $T^n(e_t(i))$. Under the Chinese taxation system, labor income, such as wages and salaries are applicable to the progressive tax rate. We let $\tau_{i,t}^s(m)$ be the personal income tax rate for family i with income m at the current period and it satisfies

$$\tau_{i,t}^s(m) = \int_0^m \tau_{i,t}^s(m')dm' \tag{3}$$

Then we write the Euler equation of the heterogeneous family as

$$c(x;S)^{-1} = \beta_1 \mathbb{E}\left\{(1+r(S'))c\left[(1+r(S'))o+in';S'\right]\right\}^{-1} \tag{4}$$

(2) Capital goods holders

Capital goods holders and heterogeneous families have the same utility function. We distinct capital goods holders from heterogeneous families to isolate the wealth accumulation effect of capital. Capital goods holders own capital k_t,

DOI: 10.24818/18423264/55.1.21.04

invest in risk-free bonds b_t at nominal value, choose optimal consumption c_t and labor time l_t to maximize the following lifetime utility:

$$\max \sum_{t=0}^{\infty} \beta_2^t \left(\log c_t - \varphi_1 \frac{l_t^{1+\varphi_2}}{1+\varphi_2} \right) \tag{5}$$

$$\begin{aligned} s.t. \quad & (1+\tau_c) c_t + k_{t+1} + \tilde{b}_{t+1} p_{t+1} - \tilde{b}_t = \\ & I_{t-1} \tilde{b}_t + d_t + w_t l_t - \tau_t^s + \left(1 + (1-\tau_k) r_t\right) k_t - \xi / 2 \left(\Delta k_{t+1} / k_t\right)^2 k_t + T_t^a \end{aligned} \tag{6}$$

Where the subjective discount factor of capital goods holders is $\beta_2 \in (0,1)$, $\beta_2 > \beta_1$. Because capital goods holders do not have the individual unemployment risk, they will be more patient. φ_1 and φ_2 represent the willingness to work and Frisch labor supply elasticity, respectively. In the constraint function, we define $\tilde{b}_t = b_t / p_t$ and $\pi_t = p_t / p_{t-1}$. The left side of constraint function shows how funds are used, where τ_c is the consumption tax rate. The right side shows that the funds come from the nominal interest rate of the bonds $I_{t-1} \tilde{b}_t$, the dividends d_t obtained from the intermediate goods manufacturers, and the labor income $w_t l$, where w_t is the average wage. τ_t^s represents a sum tax levied on the income of capital goods holders, τ_k is the capital tax rate, T_t^a is a one-time transfer payment to capital goods holders, r_t represents the rate of return on capital, and ξ is the parameter of capital adjustment costs.

(3) Firms

We refer to Blanchard and Gali (2010) and introduce price stickiness in the production sector. We assume that the final product manufacturers are in a completely competitive market, and the intermediate product manufacturers are in a monopolistic competition. The final product manufacturer produces a combination of intermediate products by

$$y_t = \left[\int_0^1 y_t(j)^{(\theta_t - 1)/\theta_t} dj \right]^{\theta_t/(\theta_t - 1)} \tag{7}$$

Where θ_t is the substitution elasticity of intermediate products $y_t(j)$. The final product manufacturer maximizes profits by choosing the best amount of $y_t(j)$:

$$\Pi_t = p_t \left[\int_0^1 y_t(j)^{(\theta_t - 1)/\theta_t} dj \right]^{\theta_t/(\theta_t - 1)} - p_t(j) \int_0^1 y_t(j) dj \tag{8}$$

We can get the demand function of the intermediate goods manufacturer and the aggregate price function of the final goods manufacturer:

$$y_t(j) = \left(p_t(j) / p_t\right)^{-\theta_t} y_t, \quad p_t = \left[\int_0^1 p_t(j)^{1-\theta_t} dj \right]^{1/(1-\theta_t)} \tag{9}$$

According to the Calvo (1983) sticky pricing mechanism, we set the proportion of intermediate goods manufacturers to adjust prices as ϕ in each period. Intermediate manufacturers choose optimal prices p_t^*, output of intermediate manufacturers $y_s(j)$, capital and labor invested by intermediate manufacturers $\left\{k_s(j), l_t^f(j)\right\}$ to maximize profits. The optimization problem of intermediate product manufacturers is given by

$$\max \mathbb{E}_t \left[\sum_{s=t}^{\infty} \lambda_{t,s} (1-\phi)^{s-t} (1-\tau_d) d_{t+s}(j) \right] \tag{10}$$

DOI: 10.24818/18423264/55.1.21.04

$$\begin{aligned} s.t.\quad & y_s(j)=\left(p_t^*/p_s\right)^{-\theta} y_s \\ & y_s(j)=a_s k_s(j)^{\alpha} l^f(j)^{1-\alpha} \\ d_{t+s}(j)=&\left(p_t^*/p_t\right) y_s(j)-w_s l_s^f(j)-\left(r_s+\delta\right)k_s(j)-F \end{aligned} \tag{11}$$

Where $\lambda_{t,t+s}$ is the random discount parameter[1], and $\lambda_{t,t+s}=\beta_1^s c_t/c_{t+s}$. δ denotes the capital depreciation rate, F indicates fixed assets, $d_{t+s}(j)$ is the profits of intermediate goods manufacturers, and τ_d is the corporate income tax rate. The process of finding the overall price evolution is given by

$$p_t=\left((1-\phi)\,p_{t-1}^{1-\theta}+\phi\left(p_t^*\right)^{1-\theta}\right)^{1/(1-\theta)} \tag{12}$$

(4) Government

Government levies consumption tax, labor tax and capital income tax on capital goods holders and heterogeneous families, make ordinary transfer payments to all households, and provide unemployment and poverty subsidies to heterogeneous families. Government departments make productive fiscal expenditures and hold government bonds. The fiscal balance equation is

$$\left\{\begin{array}{l}\tau_c\left(\int_0^{\varpi} c_t(i)\,di+c_t\right)+\tau_t^s+\int_0^{\varpi}\tau_t^s(i)\,di+\tau_k k_t+\\ \tau_d\int_0^1 d(j)dj-T_t^a-T_t^b-\int_0^{\varpi}\left[T_t^u(i)+T_t^n(i)\right]di\end{array}\right\}=g_t+\left(1+I_{t-1}\right)\tilde{b}_t-\tilde{b}_{t-1} \tag{13}$$

Referring to Gali (2008) and Leeper (2010), we set the government expenditure and the general transfer payment function for capital goods holders as follows, and both choose simple linear rule forms to.

$$\log\left(g_t\right)=\log\left(\bar{g}\right)-\gamma^g\log\left(\frac{B_t/p_t}{\bar{B}}\right) \tag{14}$$

$$T_t^a=\bar{T}^a+\gamma^T\log\left(\frac{B_t/p_t}{\bar{B}}\right) \tag{15}$$

Where γ^g represents the coefficient of response of fiscal expenditure to debt, and γ^T represents the debt repayment coefficient. The larger the coefficient, the faster the debt repayment rate. $\bar{g}$ and $\bar{B}$ are the government's productive fiscal expenditure and bond quantity at steady state, respectively. The general transfer payment $\bar{T}^a$ is zero at steady state.

We set the monetary policy rules to obey the Taylor rules:

$$\frac{I_t}{\bar{I}}=\left(\frac{I_{t-1}}{\bar{I}}\right)^{\rho_r}\left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\kappa_\pi}\left(\frac{y_t}{\bar{y}}\right)^{\kappa_y}\right]^{1-\rho_r} e_t^{\varepsilon_r} \tag{16}$$

Where ρ_r is the interest rate smoothing parameter. We let the feedback of inflation to the nominal interest rate $\kappa_\pi>1$ and the feedback of expenditure to the nominal interest rate $\kappa_y\geq 0$. $e_t^{\varepsilon_r}$ subjects to the AR (1) process.

(5) Shocks and market clearing

[1] Because the capital goods owner owns the intermediate goods manufacturer, its random discount factor is the same as the capital goods owner.

DOI: 10.24818/18423264/55.1.21.04

In addition to the idiosyncratic unemployment risk of households, there are three exogenous aggregate shocks hitting the economy: technology $\log(a_t)$, markups θ_t, and monetary policy ε_r. We assume that all aggregate shocks follow the random process of AR (1). At equilibrium, the labor market, bond market, capital market, and dividend clearing are as follows.

$$\int_0^1 l_t^f(j)dj = \int_0^{\varpi} l_t(i)di + l_t \tag{17}$$

$$B_t / p_t = \int_0^{\varpi} \tilde{b}_t(i)di + \tilde{b}_t \tag{18}$$

$$k_t = \int_0^1 k_t(j)dj \tag{19}$$

$$d_t = \int_0^1 d_t(j)dj \tag{20}$$

3. Algorithm, parameter calibration and Bayesian estimation

（1） Algorithm

To solve a Hank model, the classical algorithm of Krusell and Smith (1998) uses partial information (the first-order moment) to transform the infinite dimension of the distribution into finite dimensions, and iterates the resident policy function until convergence. The Reiter (2009) algorithm linearizes the model with overall shock while maintaining the system nonlinearity with individual idiosyncratic shocks, which can include high-dimensional features of the cross-sectional distribution of state variables. For models with a large number of state variables, the Reiter (2009) algorithm is more applicable. We introduce the endogenous grid point method (Carroll, 2006; Hintermaier & Koeniger, 2010) in Reiter's algorithm(2009), the hybrid projection and perturbation algorithm, to obtain the steady state value of savings and labor supply, which is used as the initial guess value of the policy function. The specific process is as follows.

Step 1: We use the spline method to approximate the individual decision functions (savings and labor supply policy functions) and asset distribution with finite parameters, and to discretize the assets of residents in different employment states.

Step 2: Calculate the steady-state value of the model's variables with only individual idiosyncratic risks.

- First, the external loop guesses the steady-state value of families' income and tax rate, and solves the equilibrium interest rate $\bar{r} = 1/\beta_1 - 1$ from the Euler equation. Calculate the marginal output of capital at steady state $\bar{m} = 1 - 1/\theta$, and obtain the capital-labor ratio $\bar{k} / \bar{l}^f = ((r+\delta)/(\bar{m} * \alpha))^{1/(\alpha-1)}$. Solve the equilibrium wage according to the labor optimization of intermediate goods manufacturers $\bar{w} = \bar{m}(1-\alpha)(\bar{k} / \bar{l}^f)^{\alpha}$.
- Second, based on the price $\{\bar{r}, \bar{w}\}$, the inner circle calculates the parameter matrix $\mathbf{o}^*$ of the capital goods holder's approximate equation of savings $\hat{O}(x; \mathbf{o}^*)$, so that the following approximate polynomial meets the capital goods holder's Euler equation at the node, that is,

DOI: 10.24818/18423264/55.1.21.04

$$\hat{c}(x;S)^{-1} = \beta_2 \sum_{j=1}^{n_f} \left\{ \Theta_j^f \left(1 + r(f,z)\right) \hat{c}\left(X_{t,i};S'\right)^{-1} \right\} + \eta_{t,i} \qquad i = 0,...,n_x$$

Where n_x represents the number of asset nodes. there are n_x+1 nonlinear equations in the discrete system. The parameter matrixm $\mathbf{o}^*$ has n_x+1 elements, which are solved by the quasi-Newton method.

- Third, according $\left\{\bar{r}, \bar{w}, \bar{O}(x;\mathbf{o}^*)\right\}$, we calculate the transfer dynamics of the impatient family's asset distribution $\omega_{xx'}$ and the finite parameters of the invariant asset distribution $f_t(x)$ under steady state $\mathbf{f}^*$ (the non-zero solution), and obtain the asset distribution of the heterogeneous family. At the same time, we calculate the assets and income of patient families through a representative model.
- Finally, we use the results above to update the capital holder's estimated household income, and iteratively loop through step 2 until convergence.

Step 3: Linearize the approximate equation near steady state by automatic differentiation. [2]

Step 4: Calculate the first-order disturbance under the exogenous shocks.

（2） Calibration

We perform the following parameter calibration to make the model more suitable for the Chinese economy. For the static parameter calibration of the family sector, we refer to Finocchiaro and Heideken (2013) and let the subjective discount factor of heterogeneous families β_1 be 0.97. According to Zhu Jun et al.(2018), the calibrated work willingness parameter φ_1 is 2.48. Most of the existing studies take the value of Frisch labor supply elasticity φ_2 as 2 (Zhu Jun et al., 2018). Accoding to Mckay and Reis (2016), the ratio of heterogeneous households to capital goods holders ϖ is calibrated to 4. Referring to Zhang Jie et al. (2018) on the accounting standard setting and calculation of transfer payments, we let the ratio of unemployment subsidies[3] to the entire transfer payment[4] T_t^u be 0.162, and ratio of the poverty allowance[5] to the entire transfer payment T^n be 0.105. Unemployment subsidy upper limit ratio ι is calibrated according to the latest regulations of unemployment insurance, that is, unemployment insurance is paid in accordance with 70% of the local minimum wage standard for unemployed employees during the same period, and the calibration value is 0.7. For the parameters of capital goods holders, refer to Mckay and Reis (2016), we set the speed parameter of capital adjustment cost ξ be 6, the subjective discount factor of capital goods holders β_2 is calibrated to 0.989.

[2] The total variables in the system are linear, while the variables are non-linear. The linearization algorithm is the same as Sims (2002).

[3] Zhang Jie et al. (2018) use “Statistical Yearbook” for 1995-2015 unemployment insurance accounted for the total output for this calibration.

[4] Referring to Jia Junxue and Guo Qingwang (2012), fiscal expenditure includes productive fiscal expenditure and transfer payments. According to the accounting standards of Zhang Zuomin (2013), transfer payments include medical expenditures, social security and employment expenditures, cultural and sports expenditures, media expen ditures, education expenditures, pensions and social welfare relief expenses, and the sum of various subsidies.

[5] Zhang Jie et al. (2018) used the average value of the ratio of the sum of subsidies and subsidies related to em ployment and total output in the Social Security and Employment Expenditure of Statistical Yearbook 2015 from 1 995 to 2015 for this calibration.

DOI: 10.24818/18423264/55.1.21.04

For the tax rate, referring to studies by Bai Zhonglin et al. (2019), Zhang Jie et al. (2018), we set the consumption tax rate τ_c to 0.122 and the capital tax rate τ_k to 0.086. The personal income tax rate τ^s is set to 0.079 through the ratio of the personal income tax paid by urban residents to the average tax base. According to the tax law, we calibrate the corporate income tax rate τ_d to 0.25.

By averaging the contribution rate of total capital formation to GDP growth from 1998 to 2018 by the National Bureau of Statistics, we set the elasticity of capital output to 0.463. Drawing on the studies of Ma Yong (2017), in the sticky pricing mechanism, the price adjustment ratio of intermediate product manufacturer ϕ every period is calibrated to 0.24. Referring to Bian Zhonglin et al. (2019), according to the depreciation rate of social public capital, we set the value of capital depreciation rate δ to 0.0125. Consistent with Mckay and Reis (2016), the fixed cost of production F is calibrated to 0.575. Referring to studies by Bian Zhishu et al. (2019), when the manufacturer sector is at a steady state, the price mark-up is about 11%, so we set the intermediate product substitution elasticity θ to 10. By calculating the average value of the ratio of government productive fiscal expenditure[6] to nominal total output in China from 2008 to 2018, the ratio of government productive fiscal total output to total output under steady state $\bar{G}/\bar{Y}$ is calibrated to 0.17. By calculating the average value of the ratio of China's debt balance to nominal total output in 2008-2018, we set the actual balance of bonds as a percentage of total output under steady state $\bar{B}/\bar{Y}$ as 0.17.

（3） Bayesian estimation

The model includes three exogenous shocks: technology shock, markup shock and monetary shock. We use Bayesian estimation method to estimate the dynamic parameters of the shock. Specifically, through the Markov chain Monte Carlo simulation method, we conduct 200,000 samplings to obtain the posterior mean and 90% confidence interval. To avoid singularity, the number of observable variables in the model cannot exceed the number of exogenous shocks. We select real GDP, total real consumption, and nominal interest rate[7] as observation variables, and all data are processed by HP filtering. The Bayesian estimation results are shown in Table 1.

Table 1. Bayesian estimation results

Parameter	Prior distribution	Prior mean	Post mean	90% confidence interval
γ^g	normal	-0.05	-0.064	[-1.034, -0.032]
γ^T	normal	0.05	0.435	[0.421, 0.465]
ρ_r	beta	0.50	0.499	[0.497, 0.501]
κ_π	gamma	1.50	1.917	[1.906, 1.927]
κ_y	gamma	0.50	0.569	[0.554, 0.586]
ρ^{ε_r}	beta	0.50	0.895	[0.885, 0.907]

[6] According to Zhang Zuomin (2013), government productive fiscal expenditures include government nominal consumption, capital construction expenditures, and support for rural production expenditures.

[7] The data comes from the wind and Taozha databases, and the proxy variable for the nominal interest rate is the 7-day interbank borrowing rate between banks.

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ρ_a	beta	0.50	0.915	[0.905, 0.925]
ρ_θ	beta	0.50	0.477	[0.477, 0.478]
σ^{ε_a}	Inv gamma	1	0.601	[0.579, 0.610]
σ^{ε_r}	Inv gamma	1	0.031	[0.018, 0.043]
$\sigma^{\varepsilon_\theta}$	Inv gamma	1	0.164	[0.151, 0.175]

The estimation of employment-state transition probability matrix is an important part of parameter calibration, which is determined by the unemployment rate and unemployment duration. Based on the 2007-2017 average urban registered unemployment rate and the weighted average of urban unemployed people's unworked hours, the unemployment rate and unemployment duration are determined to be 0.04 and 1.89 years, respectively. Combined with relevant studies by Shuai Jing et al.(2017), the poverty return rate is determined to be 0.13 and the poverty relief rate is determined to be 0.66. At the same time, we assume that residents cannot directly transfer from the state of employment to the state of poverty, and cannot transfer from the state of poverty to the state of unemployment, that is, the transition probability of these two states a_{en} and a_{nu} in the transition probability matrix is 0. The final calculation of the employment-state transition probability matrix is given by:

$$\begin{matrix} \\ employed \\ unemployed \\ poor \end{matrix} \begin{matrix} \begin{matrix} employed & unemployed & poor \end{matrix} \\ \begin{pmatrix} a_{ee} & a_{eu} & a_{en} \\ a_{ue} & a_{uu} & a_{un} \\ a_{ne} & a_{nu} & a_{nn} \end{pmatrix} \end{matrix} = \begin{pmatrix} 0.970 & 0.030 & 0 \\ 0.435 & 0.435 & 0.130 \\ 0.660 & 0 & 0.340 \end{pmatrix}$$

4. Saving decisions and inequality

First, we need to clarify how income uncertainty caused by employment risks creates inequality. This section portrays residents' responses in a partial equilibrium. By keeping prices and expectations unchanged at steady state, we discrete uncertain process to calculate residents' savings decisions and obtain residents' stable asset distribution. At the same time, to explore influence channels of specific fiscal policy instruments, this section also conducts counterfactual experiments: reduce transfer payments, reduce proportional taxes, and reduce income tax by 10%.

(1) Optimal savings decision

From a static equilibrium point of view, we intuitively understand that if heterogeneous households have no idiosyncratic risks or they can be fully insured, the optimal savings are the same as traditional neoclassical models, that is, the supply of savings is completely elastic. However, due to considering individual unhedged idiosyncratic risks, the infinite elasticity of capital supply in a fully efficient market no longer holds. At this time, there is a corresponding change in household savings in the face of changes in interest rates, that is, heterogeneous residents have a precautionary saving motive and are willing to hold bonds at a lower interest rate. Under the dynamic state, heterogeneous households' the optimal savings rules under different fiscal policies are shown in Figure 2.

DOI: 10.24818/18423264/55.1.21.04

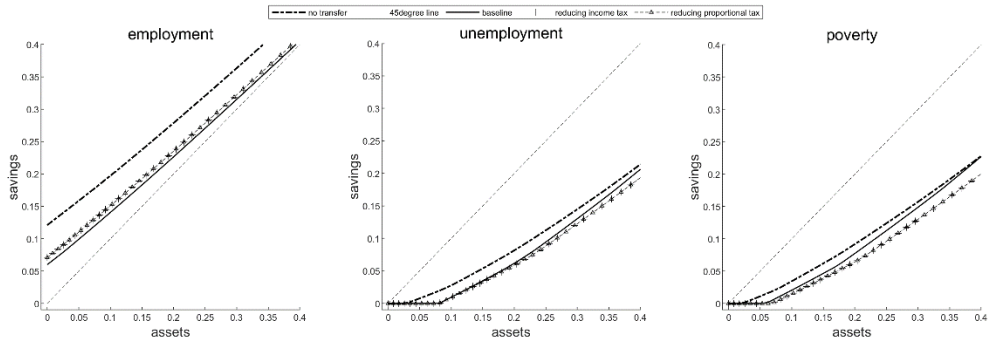

Figure 2. Optimal saving decision of heterogeneous residents

Horizontally, in the baseline scenario, employed residents have much more savings than those of unemployed and poor residents because they have no wage income. However, due to the lack of wage income, residents in unemployment and poverty states will continue to consume their minimum guaranteed income, including general transfer payments, unemployment and poverty subsidies. We can see that because the saving rate of employed residents is much higher than that of unemployed and poor residents, moreover, unemployed and poor residents have greater precautionary saving motives due to greater liquidity constraints, the widening income gap will significantly increase the household's total savings rate.

From a longitudinal perspective, we can compare the impact of three fiscal policies, which is reduction of transfer payments, income tax reduction and proportional tax reduction, on residents' optimal savings decision. The results show that reducing income tax has a certain impact on the optimal saving decision of heterogeneous residents, but the effect is weaker than transfer payment, mainly because the progressiveness of the income tax rate of personal income of Chinese residents is weak. Reducing the proportional tax rate increases residents' consumption and increases savings by increasing their actual purchasing power, but the effect is not significant compared to the baseline situation.

Reducing transfer payments has the greatest impact on heterogeneous residents' optimal savings decisions. Regardless of employment status, the savings of heterogeneous residents after transfer payments have increased. Because transfer payments are a component of the income of heterogeneous residents, especially as the entire income of unemployed and poor residents, reducing payment transfers reduces the actual income of residents. Due to liquidity constraints, heterogeneous residents will react to temporary income changes. In order to smooth consumption, heterogeneous residents will increase precautionary saving motivation (Aculert, 2019).

(2) Distribution of assets (wealth) of heterogeneous residents

Through the above optimal savings rules and asset and labor prices of heterogeneous residents, the transfer dynamics of asset distribution can be calculated, thereby obtaining the constant asset distribution of heterogeneous households. The unemployed individual's unemployment risk creates a gap between rich and poor. Since reducing the income tax and reducing the proportional tax on residents has a

DOI: 10.24818/18423264/55.1.21.04

weak influence on the optimal saving decision of heterogeneous residents, it also has a weak influence on the asset distribution of heterogeneous residents. Here we focus on reducing the impact of transfer payments on the distribution of heterogeneous residents' assets. The distribution of assets held by employed, unemployed and poor residents is shown in Figure 3.

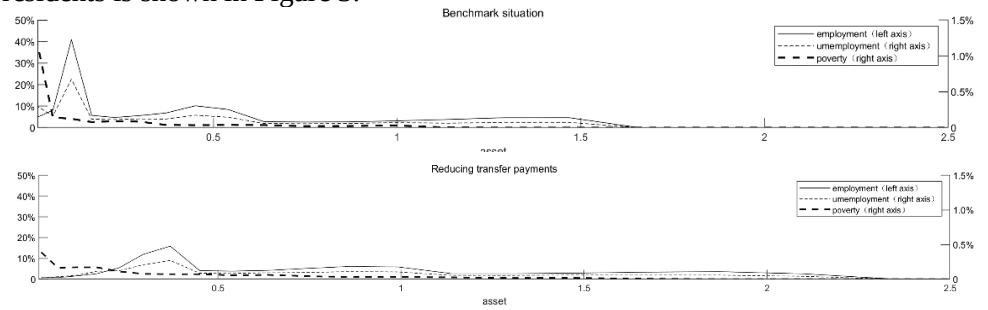

Figure 3. Asset distribution of heterogeneous residents

Under the baseline situation, residents hold assets under both employment and unemployment states. Employed residents have wage income and precautionary savings motives to mitigate the impact of idiosyncratic unemployment risks. As a result, the increased precautionary savings form part of the assets. Unemployed residents can be re-employed and earn wage income in the short term, as well as general transfer payments and unemployment subsidies, so they will hold some assets. However, most of the residents in poverty are holding assets close to zero, that is, no wage income and no assets. They rely on only poverty subsidies to survive.

From figure3, the asset distribution of heterogeneous residents shows a left-biased characteristic, that is, the population density of low assets is larger, and the population density of high assets is smaller, but the distribution of double humps reflects the low population density of middle-income residents.

By reducing transfer payments to residents, the asset distribution of employed and unemployed residents is relatively right-biased, and the density of poor residents without any assets is relatively reduced. Through the above analysis of the heterogeneous residents' optimal savings decision, we can see that reducing transfer payments to heterogeneous residents make the precautionary savings channels play a major role. Heterogeneous residents will increase their savings thereby increasing the assets they hold. In addition, because the reduction in transfer leads to a reduction in residents' actual disposable income, employed residents will be more motivated to increase their workload. At this time, the marginal incentive effect will increase the employment income and assets of employed residents.

5. The impact of inequality on policy transmission

To better understand the interactive effects of inequality and policy, we examine the impulse response of key aggregates in the economy under the three exogenous shocks, which are total technology shock, mark-up shock and monetary shocks. In addition, we also focus on analysing the impact of fiscal policy shocks on economic output.

DOI: 10.24818/18423264/55.1.21.04

(1) The impact of monetary policy

In a representative agent model, the economic agent is essentially a permanent income consumer. The transient income changes do not affect their behavior, that is, income or wealth inequality does not affect the economic agent's behavior, which is poorly fitted to empirical results. However, from the above analysis, we can see that in the heterogeneous agent model, due to the difference between residents' preventive savings motives and marginal consumption propensity affect total savings and total consumption. Therefore, the effect of policy transmission will be affected by the distribution of income and wealth. The impulse response of the main economic variables to the three overall shocks of technological shock, price shock and currency shock is shown in Figure 4.

In the short term, a positive standard deviation of technical shocks, currency shocks, and cost-plus price shocks will cause 0.43%, 0.31%, and 0.14% output increases, and 0.32%, 0.14%, and 0.13% consumption increases. The volatility of consumption is less than the volatility of output, reflecting the increase in residents' savings, which shows to a certain extent that residents have preventive savings motives.

In the representative agent model, the impact of monetary policy on consumption is mainly through intertemporal substitution. However, in the heterogeneous agent model, because of the unhedged individual idiosyncratic risks, the gap between the rich and the poor exists. Residents are not sensitive to changes in interest rates brought by monetary policy, but are more sensitive to income shocks. The positive monetary policy stimulates consumption mainly through expanding the labor market, increasing labor demand and labor income (Kaplan et al, 2018).

Figure 4 shows that on the one hand, one unit positive monetary shock significantly increases working hours to 0.7%, thereby increasing the working income of employed residents. On the other hand, a positive currency shock pushes inflation up by 0.62% and asset prices by 1.3%. Unemployed and poor families have no wage income, and the vast majority of poor residents hold zero assets and cannot enjoy the wealth effect of monetary policy stimulus. Therefore, compared with unemployed and poor families, employed families are relatively wealthier. The relatively affluent employed households have a low marginal propensity to consume while unemployed and poor households have greater willingness to consume but have no spending power. The expansion of income gap will weaken the consumption stimulus of monetary policy effect.

In addition, the transmission of monetary policy is also more dependent on fiscal channels. Changes in interest rates caused by monetary policy affect the government's intertemporal budget constraints, and the government responds with some form of fiscal means, which affect the family's balance sheet and thus the overall economy when the Ricardo equivalence is no longer hold. Figure 4 shows that a unit of positive monetary policy stimulus will cause government purchases to increase by 0.6% and government debt to decrease by 0.5% in the short term. It means if monetary policy adopts short-term interest rate reduction stimulus, government debt expenditure can be reduced and converted into fiscal stimulus, which will affect the economic aggregate.

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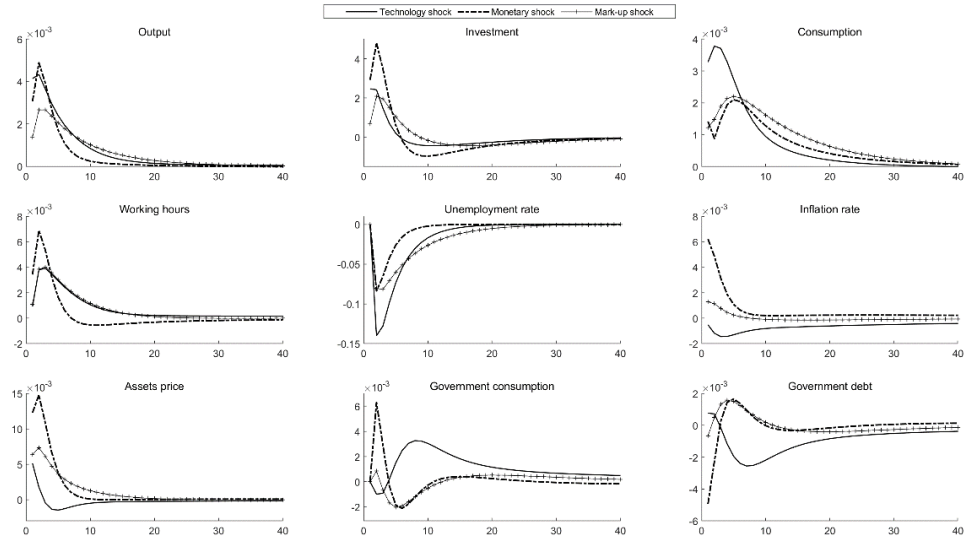

Figure 4. Impulse response of the overall shock

(2) The impact of fiscal policy

In order to understand the effect of fiscal policy on the economic aggregate in the heterogeneous agent model, we calculate the government productive fiscal expenditure multiplier, income tax multiplier and transfer payment multiplier, which are 0.81, 0.26 and 0.40, respectively, which are more significant than those of the representative agent model. Under the representative agent model, the effect of productive fiscal expenditure multiplier is generally based on the two mechanisms: one is that output affects aggregate demand, another is price rigidity. However, under our heterogeneous agent model, the permanent income assumption no longer holds. A temporary increase in household income from policy stimulus will increase MPC and weaken precautionary savings motives, thereby stimulating consumption, increasing total output, and increasing labor supply. As the income of residents increases, the multiplier effect of consumption increases. In general, the fiscal multiplier has a significant indirect effect by affecting residents' precautionary savings and marginal consumption propensity, thereby affecting total economic output.

In addition, if the active fiscal policy targets low-income poor households with higher MPCs, such as increasing transfers and subsidies for unemployed and poor households, it will not only help narrow the gap between rich and poor, but also relax their liquidity constraints and substantially stimulate consumption and output.

6. Conclusions

We construct a Hank model and study the role of fiscal policy in reducing inequality through counterfactual experiments. Conclusions are: (1) The saving rate of employed residents is much higher than that of unemployed and poor residents. Low-income households are subject to higher liquidity constraints, and precautionary savings will significantly increase the savings rate. Therefore, the

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increase in inequality will increase the total savings rate, which is not conducive to economic transformation. (2) The assets of residents are characterized by dumbbells. The general payment transfer has no significant effect on improving income inequality. The impact of income tax on inequality depends on progressive efficiency, and reducing the proportional tax can improve inequality to a certain extent. (3) Special unemployment and poverty subsidies can improve inequality through the disposable income effect. Increasing the income of residents with high MPC will significantly increase consumption and total demand. We can achieve a balance between fairness and efficiency. (4) The improvement of income inequality makes the transmission of monetary policy more effective in stimulating consumption. The effectiveness of monetary policy depends on the fiscal response.

Empirical facts show that the uneven distribution of wealth among residents is greater than the uneven distribution of income. Future research can focus on the inequality of wealth, starting from the heterogeneity of assets held by residents to study the mechanism behind income and wealth inequality, and the effects of monetary policy, fiscal policy and policy coordination.

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